ECE 330 Exam 1: Summer 2019 NAME Solution
90 minutes J. Schuh

1. __________/25 2. __________/25

3. __________/25 4. __________/25 TOTAL _________/100

USEFUL INFORMATION

sin(x)=cos(x-90°) $\bar{V} = \bar{Z}\bar{I}$ $\bar{S} = \bar{V}\bar{I}^* = P + jQ$ $\bar{S}_{3\varphi} = \sqrt{3}V_L I_L \angle\theta$

0<θ<180° (lag) $I_L = \sqrt{3}I_\varphi$ (delta) $\bar{Z}_Y = \bar{Z}_\Delta/3$

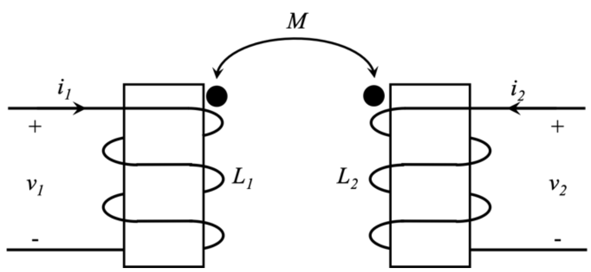
-180°<θ<0 (lead) $V_L = \sqrt{3}V_\varphi$ (wye) $\mu_0 = 4\pi \times 10^{-7}$ H/m

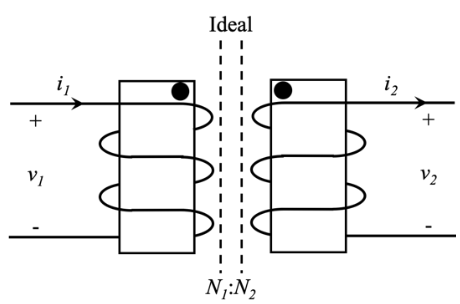
ABC phase sequence has A at 0, B at -120°, and C at +120°

$\int \underline{H} \cdot \underline{dl} = \int \underline{J_f} \cdot \hat{n} dA$ $\int \underline{E} \cdot \underline{dl} = -\frac{d}{dt}\left(\int \underline{B} \cdot \hat{n} dA\right)$ $\mathcal{R} = \frac{l}{\mu A}$ $Ni = \mathcal{R}\varphi$

$\varphi = BA$ $\lambda = N\varphi = Li$ (if linear) $v = \frac{d\lambda}{dt}$ $k = \frac{M}{\sqrt{L_1 L_2}}$

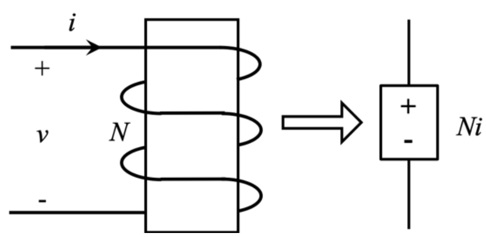
1hp=746 W





$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$ $a = \frac{N_1}{N_2}$ $N_1 i_1 = N_2 i_2$

$v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$ $\frac{V_1}{V_2} = \frac{N_1}{N_2}$



Problem 1 (25 Points)

4 single-phase loads are connected in parallel to a generator that supplies 480 V. The loads are given as:

1) A purely resistive load that consumes 50 kW
2) A load that consumes 10 kVA at a power factor of 0.707 leading
3) A load with impedance $\underline{Z}$=120+j50 Ω
4) A purely inductive load of 30 kVAR

a) What is the total power supplied by the source? (12 points)

$\bar{S}_1 = 50 + j0$ kVA

$\theta_2 = -\cos^{-1}(0.707) = -45°$

$\bar{S}_2 = 10\angle -45°$ kVA

$= 7.071 - j7.071$ kVA

$\bar{Z}_3 = 120 + j50\ \Omega$

$= 130\angle 22.620°\ \Omega$

$\bar{S}_3 = \frac{V^2}{Z}\angle 22.620$

$\bar{S}_3 = 1.772\angle 22.620°$ kVA

$= 1.636 + j0.682$ kVA

$\bar{S}_4 = 0 + j30$ kVA

$\bar{S}_{tot} = \bar{S}_1 + \bar{S}_2 + \bar{S}_3 + \bar{S}_4$

$\bar{S}_{tot} = 58.706 + j23.610$ kVA

$= 63.276\angle 21.908°$ kVA

b) What is the source current? (5 points)

$\bar{S}_{tot} = \bar{V}\bar{I}_s^*$

$\bar{I}_s = \left(\frac{\bar{S}_{tot}}{\bar{V}}\right)^*$

$\bar{I}_s = 131.824\angle -21.908°$ A

c) A capacitor is connected in parallel with the loads to make the power factor 0.995 lagging. What is the magnitude of the new source current? (8 points)

$\theta_n = \cos^{-1}(0.995)$

$= 5.732°$

$Q_n = P\tan(\theta_n)$

$Q_n = 5.893$ kVAR

$S_n = \sqrt{P^2 + Q_n^2}$

$S_n = 59.001$ kVA

$I_n = \frac{S_n}{V}$

$I_n = 122.919$ A

Problem 2 (25 points)

Three 3-phase loads are connected in parallel to a wye-connected source with 208 V line to line. The loads are given as:

1) A wye connected load consuming 10 kVA at a 0.95 leading power factor
2) A wye connected load consuming 80 kW at a 0.975 lagging power factor
3) A delta connected load with impedance *Z*=15+j8 Ω

a) What is the source line current? (13 points)

$\theta_1 = \cos^{-1}(0.95)$

$\theta_1 = -18.195°$

$\bar{S}_1 = 10\angle -18.195°$ kVA

$= 9.5 - j3.123$ kVA

$\theta_2 = \cos^{-1}(0.975)$

$\theta_2 = 12.837°$

$\bar{S}_2 = \frac{80}{0.975}\angle 12.837°$

$\bar{S}_2 = 80 + j18.232$ kVA

$\bar{Z}_{3\Delta} = 15 + j8\,\Omega$

$= 17\angle 28.073\,\Omega$

$\bar{Z}_{3Y} = \frac{Z_{3\Delta}}{3}\angle 28.073\,\Omega$

$\bar{Z}_{3Y} = 5.667\angle 28.073°\,\Omega$

$\bar{S}_3 = 3\left(\frac{120^2}{5.667}\right)\angle 28.073°$

$\bar{S}_3 = 7.635\angle 28.073°$ kVA

$= 6.737 + j3.593$ kVA

$\bar{S}_{tot} = \bar{S}_1 + \bar{S}_2 + \bar{S}_3$

$= 96.237 + j18.703$ kVA

$= 98.037\angle 10.998°$ kVA

$\bar{S}_{tot} = \sqrt{3}\bar{V}_L\bar{I}_L^*$

$\bar{I}_L = \left(\frac{\bar{S}_{tot}}{\sqrt{3}\bar{V}_L}\right)^*$

$\boxed{\bar{I}_L = 272.124\angle -10.998°\ A}$

b) A delta connected capacitor bank is added in parallel to the three loads to make the overall power factor 0.995 lagging. What is the new source line current? (12 points)

$\theta_n = \cos^{-1}(0.995)$

$\theta_n = 5.732$

$Q_n = P\tan(\theta_n)$

$Q_n = 9.660$ kVAR

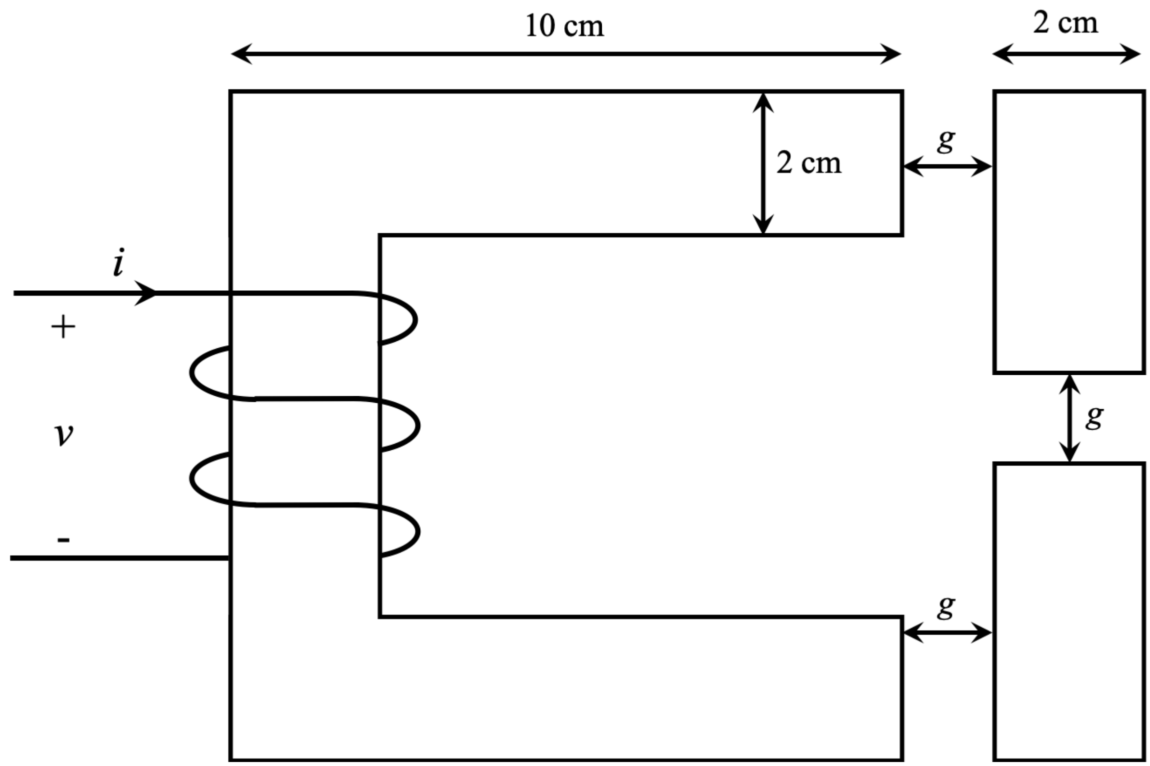
$\bar{S}_n = 96.237 + j9.660$ kVA

$\bar{S}_n = 96.720\angle 5.732$ kVA

$\bar{I}_L = \left(\frac{\bar{S}_{tot}}{\sqrt{3}\bar{V}_L}\right)^*$

$\boxed{\bar{I}_L = 268.468\angle -5.732°\ A}$

Problem 3 (25 Points)



A coil is wound 250 times around an iron core with infinite permeability (dimensions given above) that has an air gap of 5 mm and a depth into the page of 3 cm. Fringing effects are included.

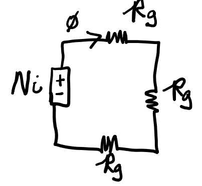
With the voltage polarity and current direction as defined above:

a) What is the area of the air gap (include fringing)? (3 points)

$A = (2cm + 5mm)(3cm + 5mm)$

$\boxed{A = 0.000875 m^2}$

b) Draw the magnetic equivalent circuit and determine the reluctance values for the air gap. (4 points)



$R_g = \frac{g}{\mu_0 A}$

$R_g = 4.547 \times 10^6$ At/Wb

c) What is the inductance of the coil? (10 points)

$Ni = \phi(3R_g)$

$\phi = \frac{N}{3R_g} i$

$\lambda = N\phi \Rightarrow \lambda = \frac{N^2}{3R_g} i = Li$

$L = \frac{N^2}{3R_g}$

$L = 0.00458 \text{ H}$

d) What is the inductance of the coil if fringing is neglected? (8 points)

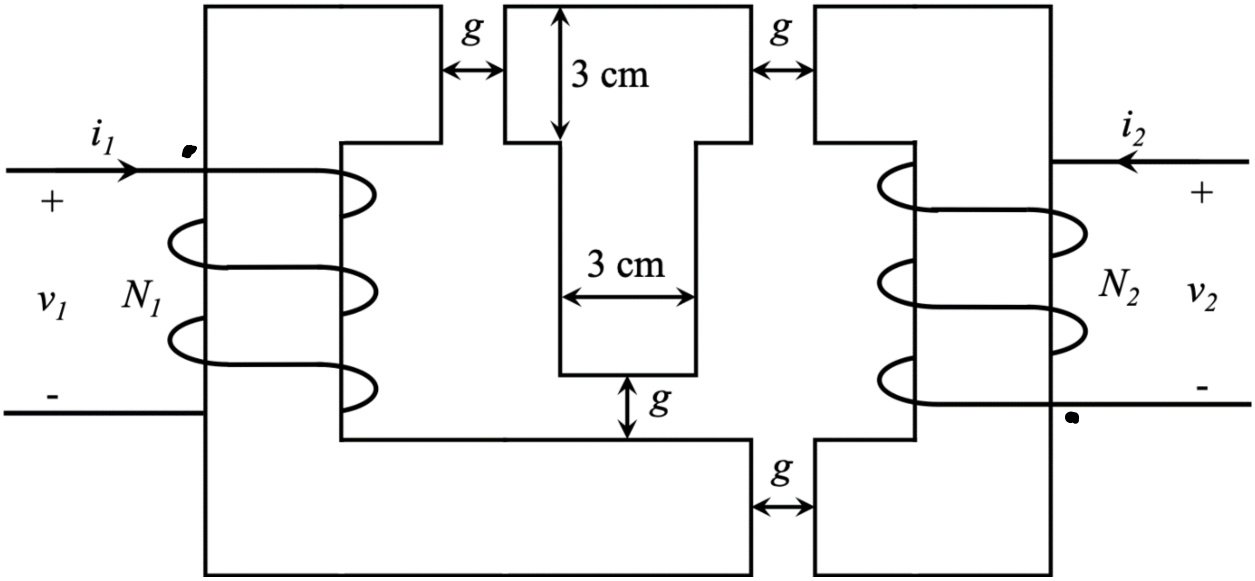
$A = 0.0006$

$R_g = \frac{g}{\mu_0 A} = 6.631 \times 10^6 \text{ At/Wb}$

$L = \frac{N^2}{3R_g}$

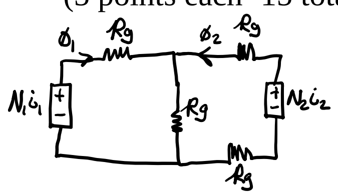
$L = 0.00314 \text{ H}$

Problem 4 (25 Points)



Two coils are wrapped around an iron core with infinite permeability as shown. Coil 1 has 400 turns while coil 2 has 100 turns. The air gap g is 2.5 mm and the depth into the page is 2 cm. Neglect fringing.

a) Put the dot markings on the two coils. (5 points)
b) What is the self-inductance and the mutual inductance of the two coils? (5 points each=15 total points)



$$R_g = \frac{g}{\mu_0 A}$$

$$A = 0.0006 m^2$$

$$R_g = 3.316 \times 10^6 \text{ At/Wb}$$

$$N_1 i_1 = \phi_1 R_g + (\phi_1 + \phi_2) R_g$$

$$N_1 i_1 = 2\phi_1 R_g + \phi_2 R_g$$

$$\phi_2 = \frac{N_1 i_1 - 2\phi_1 R_g}{R_g}$$

$$\phi_2 = \frac{N_1 i_1 - 2\left(\frac{3}{5} N_1 i_1 - \frac{1}{5} N_2 i_2\right)}{R_g}$$

$$\phi_2 = -\frac{1}{5}\frac{N_1 i_1}{R_g} + \frac{2}{5}\frac{N_2 i_2}{R_g}$$

$$\lambda_1 = N_1 \phi_1 \Rightarrow \lambda_1 = \frac{3}{5}\frac{N_1^2}{R_g} i_1 - \frac{1}{5}\frac{N_1 N_2}{R_g} i_2$$

$$\lambda_2 = N_2 \phi_2 \Rightarrow \lambda_2 = -\frac{1}{5}\frac{N_1 N_2}{R_g} i_1 + \frac{2}{5}\frac{N_2^2}{R_g} i_2$$

$$N_2 i_2 = \phi_2 (2R_g) + (\phi_1 + \phi_2) R_g$$

$$N_2 i_2 = 3\phi_2 R_g + \phi_1 R_g$$

$$N_2 i_2 = 3(N_1 i_1 - 2\phi_1 R_g) + \phi_1 R_g$$

$$N_2 i_2 = 3N_1 i_1 - 5\phi_1 R_g$$

$$5\phi_1 R_g = 3N_1 i_1 - N_2 i_2$$

$$\phi_1 = \frac{3}{5}\frac{N_1 i_1}{R_g} - \frac{N_2 i_2}{5R_g}$$

$L_1 = 0.02895$ H
$L_2 = 0.001206$ H
$M = 0.002413$ H

c) What is the RMS magnitude of the open circuit voltage on coil 2 if $i_1(t)=10\cos(377t)$? (5 points)

$$V_1 = \frac{d\lambda_1}{dt} = L_1\frac{di_1}{dt} - M\frac{di_2}{dt}$$

$$V_2 = \frac{d\lambda_2}{dt} = -M\frac{di_1}{dt} + L_2\frac{di_2}{dt}$$

Open Circuit: $i_2 = 0$

$$\frac{di_1}{dt} = -10(377)\sin(377t)$$

$$= 10(377)\sin(-377t)$$

$$= 10(377)\cos(-377t-90^\circ)$$

$$= 10(377)\cos(377t+90^\circ)$$

$$V_2 = -M\frac{di_1}{dt}$$

$$V_2 = -9.096\cos(377t+90^\circ)\,V$$

$$= 9.096\cos(377t+90^\circ-180^\circ)\,V$$

$$V_2 = 9.096\cos(377t-90^\circ)\,V$$

$$V_{2rms} = \frac{9.096}{\sqrt{2}}$$

$$\boxed{V_{2rms} = 6.432\ V}$$

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